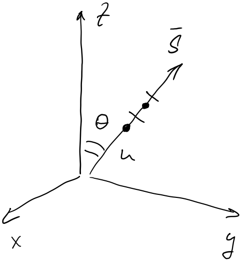
Ray velocity ellipsoid

From previous discussion we have following conclusions:



1) In crystal, there is direction, called optical axis of crystal z, along which birefringence is not observed.

2) If we look at any other direction $\bar{S}$, then along this direction we will have propagation of two waves:

Ordinary wave

$$u = \frac{c}{\sqrt{\varepsilon_\perp}}$$

Polarized $\perp$ main section

Extraordinary wave

$$u = \frac{c}{\sqrt{\varepsilon_\parallel \sin^2\theta + \varepsilon_\perp \cos^2\theta}}$$

Polarized in plane of main section

Let's look at the last formula. Let's play with it and bring into the following form:

$$\frac{u^2 \sin^2\theta}{c^2/\varepsilon_\parallel} + \frac{u^2 \cos^2\theta}{c^2/\varepsilon_\perp} = 1$$

Obviously,

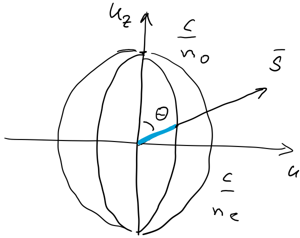
$$\begin{cases} u\cos\theta = u_z \\ u\sin\theta = u_y \end{cases} \qquad \underset{\substack{\uparrow \\ \text{ordinary}}}{n_o = \sqrt{\varepsilon_\perp}} \qquad \underset{\substack{\uparrow \\ \text{extraordinary}}}{n_e = \sqrt{\varepsilon_\parallel}}$$

The equation is then:

$$\frac{u_y^2}{\left(\frac{c}{n_e}\right)^2} + \frac{u_z^2}{\left(\frac{c}{n_o}\right)^2} = 1$$

This is ellipse equation with half-axis $\frac{c}{n_e}$ and $\frac{c}{n_o}$

Let's draw this ellipse.

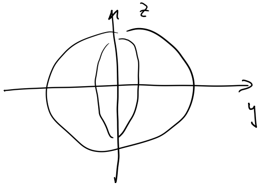


What is the meaning of this ellipse?

If in the center of coordinates we have the source that emits extraordinary wave polarized in the plane of main section, then the distance from center of coordinates to ellipse will be proportional to ray velocity. It is clear, along z the speed will be $\frac{c}{n_o}$.

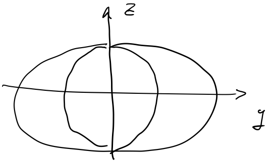
The circle demonstrates what will be the speed of ordinary wave. Since this speed is independent on θ, we have circle of radius $\frac{c}{n_o}$.

Please note, in this crystal ellipse is included in circle



Such crystals are called *positive crystals*.

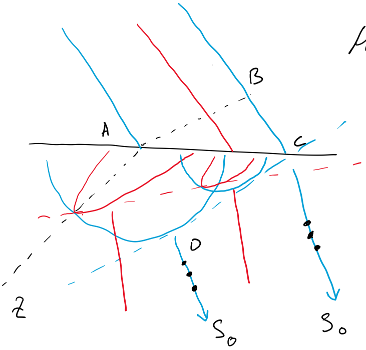
$n_e > n_o$



These is negative crystal

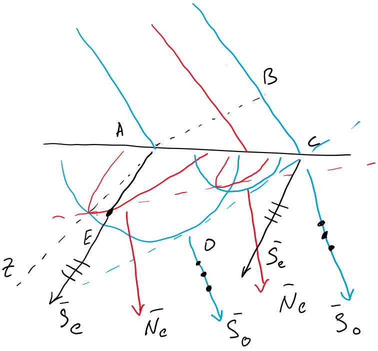
$n_e < n_o$

Huygens construction for single axis media



$n_e > n_o$

Huygens principle says that new front is an envelope of secondary waves fronts.

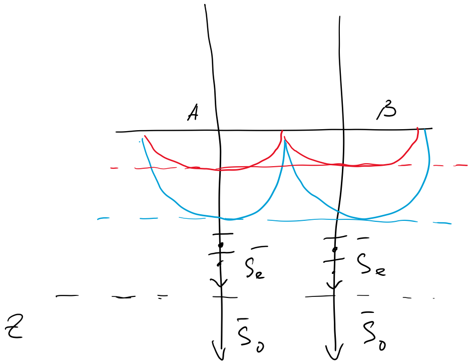


$n_e > n_o$

Vectors $\bar{S}_e$ and $\bar{N}_e$ are at angle α.

It is clear, we have we have two waves along $\bar{S}_o$ and $\bar{S}_e$.

The special case is when the crystal is cut such as light penetrate perpendicular to z.



Waves propagates in the same direction, but with different speed.

This effect can be easily utilized to produce polarized light.

Wollaston prism

Polaroids

In contrast to bulk prism and directional anisotropy, polaroids are based on dichroism, i.e. ability to absorb light of one polarization.

Demonstration